$f(x) = ax^2 - ax$
$f'(x) = 2ax - a$
$f''(x) = 2a$

No.
Date

2 (1) $f(0) = f(1) = 0$ より $f(x) = b(x^2 - x)$ (b: Const.)

$f''(x) = 2b$ より、$f''(0) = 2a$

$\therefore a = b$

$\therefore f(x) = a(x^2 - x)$

(2) $(f'(x) - x)^2 - f(x) = \{(2a-1)x - a\}^2 - a(x^2 - x)$

$(4a^2 - 5a + 1)x^2 + (-4a^2 + 3a)x + a^2$

$\therefore \int_0^1 \{(f'(x) - x)^2 - f(x)\} dx = \frac{4}{3}a^2 - \frac{5}{3}a + \frac{1}{3} - 2a^2 + \frac{3}{2}a + a^2$

$= \frac{1}{3}a^2 - \frac{1}{6}a + \frac{1}{3}$

$= \frac{1}{3}\{(a - \frac{1}{4})^2 + \frac{15}{16}\}$

$\therefore$ 与式は $a = \frac{1}{4}$ で最小となる

$\therefore$ 求める答えは $f(x) = \frac{1}{4}(x^2 - x)$ ...(ア)、で最小値 $\frac{5}{16}$ ...(イ)

(3) $I - J = \int_0^1 \{2h'(x)f'(x) - 2h'(x)x - h(x)\} dx$

$2\{f'(x) - x\} = -x - \frac{1}{2}$

$\int_0^1 h'(x) dx = 0$

$\int_0^1 h'(x) \cdot x dx = [h(x) \cdot x]_0^1 - \int_0^1 h(x) dx$

$= -\int_0^1 h(x) dx$

$\therefore \int_0^1 \{2h'(x)f'(x) - 2h'(x)x - h(x)\} dx$

$= 0$

$\therefore I - J = 0$ より $I = J$ ■

(4) $f(x) + h(x) = g(x)$ とすれば

(左辺) $= I = J = \int_0^1 [\{f'(x) - x\} - f(x)] dx + \int_0^1 (h'(x))^2 dx$

$(h'(x))^2 \geqq 0$ より $\int_0^1 (h'(x))^2 dx$

$\therefore J \geqq \int_0^1 [\{f'(x) - x\} - f(x)] dx \geqq m$

$\therefore$ 左辺 $\geqq m$ ■